1. (10%) Make a simple drawing of the Smith chart, and explain conceptually how the 200Ω loading impedance can be transform to $50+j50\Omega$ using one capacitor and one inductor?
2. (10%) A standard WR10 rectangular waveguide has dimensions of 0.1 inches × 0.05 inches, i.e., 2.54 mm × 1.27 mm. What is the respective cutoff frequency for TE_{10}, TE_{11}, and TE_{12} mode wave propagation?
3. (10%) Please explain why antenna with large aperture size (i.e., large effective physical area A_{eff}) can have fine angular resolution (i.e., small beam size Ω_A)? Or mathematically, why

$$A_{\mathrm{eff}} \cdot \Omega_A \cong 1$$

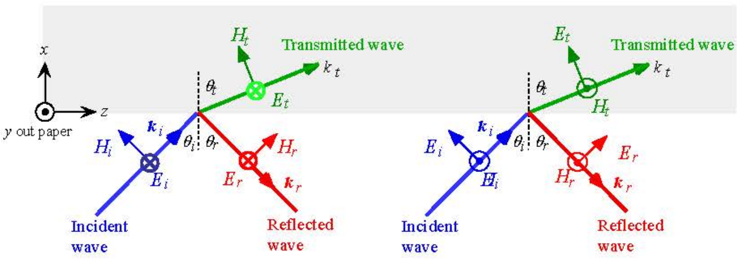
4. (15%) Use the divergence theorem $\oint_S \vec{A}\cdot n da = \int_V \nabla\cdot\vec{A} d^3x$ to show that

(a) Green's first identity: $\int_V (\phi\,\nabla^2\psi + \nabla\phi\cdot\nabla\psi)\,d^3x = \oint_S \phi\frac{\partial\psi}{\partial n}\,da$,

(b) Green's theorem: $\int_V (\phi\,\nabla^2\psi - \psi\,\nabla^2\phi)\,d^3x = \oint_S \left[\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right] da$,

(c) The general solution of the Poisson equation with specified values of the potential on the boundary surface is given by $\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o}\int_V G_D(\mathbf{r},\mathbf{r}')\,\rho(\mathbf{r}')\;d^3r' - \frac{1}{4\pi}\oint_S \Phi(\mathbf{r}')\,\frac{\partial G_D(\mathbf{r},\mathbf{r}')}{\partial n'}\,da'$.

5. (20%) (a) Derive the Snell's law. (b) Use $\mu=\mu_o$ and $n = n_2/n_1$ to derive the Fresnel's law for s and p waves from the medium with the refractive index n_1 to the medium with the refractive index n_2



(a) The electric vector of s-polarized light is perpendicular to the plane of incidence.

(b) The electric vector of p-polarized light lies in plane of incidence.

6. (10%) Due to an azimuthally symmetric charge distribution inside a sphere $(r<a)$, the potential on the positive z axis outside the sphere $(r>a)$ is

$$\Phi(z) = V\ln\left(1+\frac{a}{z}\right).$$

There is no charge outside the sphere. Find the potential $\Phi(r,\theta,\phi)$ at any point outside the sphere.

7. (15%) A dielectric sphere with electric permittivity ϵ is placed in an initially uniform electric field $\mathbf{E}_0$.

   (a) (10%) What is the electric field and the polarization within the sphere?

   (b) (5%) Describe the distribution of the polarization charge.

8. (10%) Consider the Green function for a Dirichlet problem *inside* a rectangular box defined by the six planes, $x = 0$, $y = 0$, $z = 0$, $x = a$, $y = b$, $z = c$. Show that the Green function can be expressed by

$$G(\mathbf{x}, \mathbf{x}') = \frac{32}{\pi abc} \sum_{l,m,n=1}^{\infty} \frac{\sin\left(\frac{l\pi x'}{a}\right)\sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{m\pi y'}{b}\right)\sin\left(\frac{m\pi y}{b}\right)\sin\left(\frac{n\pi z'}{c}\right)\sin\left(\frac{n\pi z}{c}\right)}{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}.$$