國立交通大學 107 學年度 暑假 博士班資格考 考試命題紙

科目：量子力學　　系所別：電子研究所　　

☐ 可看書　☑ 不可看書（不可帶計算機）　　考試日期：107 年 8 月 7、8 日

1. (12 points) Consider a particle of mass m in 1-D infinite potential well ($0<x<a$) with a small perturbation, $H'(x)=V_0 x$,
   (a) Plot the perturbed potential $V(x)$ if V_0 is a positive but small number. (2 points)
   (b) Find the eigen-energy of the n-th stationary state using time-independent perturbation theory. (10 points)

2. (10 points) Consider the following 1-D potential,
   $V(x)=\begin{cases}\alpha x, x>0\\ \infty, x\le 0\end{cases}$ where α is a positive constant.
   (a) Solve the n-th eigenenergies by using WKB approximation. (8 points)
   (b) Discuss how your result depends on α and explain its physical reason. (2 points)

3. (8 points)
   (c) Explain the triplet and singlet configurations for a two-electron system. (4 points)
   (d) Explain the bonding and anti-bonding states for a hydrogen molecule are. (2 points)
   (e) Explain why the eigenenergy of anti-bonding state is higher than that of bonding state in a hydrogen molecule. (2 points)

4. (15%) An electron of mass m is confined in a box with infinite potential well. The three box sides are of length $2L$, $2L$ and L.
   (a) (3%) Write the appropriate time-independent Schrödinger equation.
   (b) (5%) Find the lowest possible energy and write its normalized wavefunction.
   (c) (4%) Give an expression for the number of state, N, have energy less than some given E. Assume $N >> 1$.
   (d) (3%) Calculate the longest wavelength of electromagnetic radiation absorbed strongly by the electron.

5. (18%) A spin-1/2 particle is expressed in the basis of eigenstates of S_z.
   (a) (4%) Find the matrix representing S_z and S_x.
   (b) (6%) If the particle is subject to the Hamiltonian $H=S_x+S_z$, calculate the energy levels of this system.
   (c) (8%) From (b), if at time zero the spin is in an eigenstate of S with $S_z = +\hbar/2$, calculate the expectation value of the spin at time t.

6. (20%) For any state $|\psi\rangle$

$$\langle \mathbf{r}|L_x|\psi\rangle = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta}+\cos\phi\cot\theta\frac{\partial}{\partial\phi}\right)\langle\mathbf{r}|\psi\rangle$$

$$\langle \mathbf{r}|L_y|\psi\rangle = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta}+\sin\phi\cot\theta\frac{\partial}{\partial\phi}\right)\langle\mathbf{r}|\psi\rangle$$

   Using the above identities, calculate the spherical harmonics $Y_l^l(\theta,\phi)$ and $Y_l^{l-1}(\theta,\phi)$, but you don't need to carry out their normalization constants.

7. (17%) Explain the meaning of the phase shift for a particle scattered by a central potential. Then calculate the phase shift (for any l) of a hard sphere of radius r_0, and further calculate the phase shift of the l=0 explicitly. (Hint: in a free space, the radial part of a simultaneous eigenfunction of energy and L^2 is a spherical Bessel function)