Free electron mass = 9.1×10^{-31} kg; $\hbar = 1.05 \times 10^{-34}$ J/sec

1. To find the degree of space filling for a given lattice, first find the distance d_{min} between nearest neighbors. Then put circles (in two dimensions) or spheres (in three dimensions) of diameter d_{min} on each lattice site, and ask what portion of space is occupied by the circles or spheres. This number is the packing fraction.

   (a) (4%) Calculate the packing fractions for the square and hexagonal lattices in two dimensions.
   (b) (12%) Calculate the packing fractions for the following three-dimensional lattices: Simple cubic, body-centered cubic, face-centered cubic, and hexagonal close-packed.

2. (14%) Assume that a 3D metal has a simple cubic lattice with a lattice constant of 0.5 nm, and that each atom has one valence electron which becomes a conduction electron in the solid. Based on free electron model, estimate the Fermi energy for the metal.

3. Assume that E vs. k relationship for electrons in the conduction band of a n-type semiconductor can be approximated by

$$E = ak^2 + b,$$

where a and b are constants.
The cyclotron resonance for electrons in a field B = 0.1 Weber-m^{-2} occurs at an angular frequency = 1.8×10^{11} rad s^{-1}.

(a) (10%) Find the value of a.
(b) (10%) Estimate the carrier concentration in the semiconductor, given that the Hall coefficient at room temperature is $R_H = 6.25 \times 10^{-6}$ m^{-3}coul^{-1}.

4. Assuming that the crystal potential of a one-dimensional solid with lattice constant a is given by

$U(x)=aV_o \sum\delta(x-na)$, n is an integer

where V_o is a positive constant. In the one-dimensional Kronig-Penny model, the electron *E-k* relationship can be derived from the following equation

$\cos(ka)=\cos(\alpha)+P\sin(\alpha)/\alpha$, where $\alpha=(2ma^2E/\hbar^2)^{0.5}$ and $P=V_oma^2/\hbar^2$,

(a) Please calculate the maximum energy in the first band and the corresponding *k* values. (5%)
(b) Assuming V_o is sufficiently small, please calculate the **minimum** energy in the second band and the corresponding *k* values. (hint, please use Taylor expansions $\cos(\alpha)\sim -1+(\alpha-\pi)^2/2$ and $\sin(\alpha)\sim -(\alpha-\pi)$ as α is close to π.) (15%)
(c) What is the bandgap energy from (b) ? (5%)

5. Assume that an ionized impurity atom has an attracting potential as follows

$U(x)=-1eV$ for $-a<x<a$, and

$U(x)= 0eV$ elsewhere

In a one-dimensional metal, assume that a free electron has a wave-vector k and the ionized impurity scattering is elastic.

(a) What is the electron wave-vector after ionized impurity scattering ? (5%)
(b) Please calculate the ionized impurity scattering rate. Express your result in terms of k, a, π, .. (20%)